Role of Chaos in Swarm Intelligence - A Preliminary Analysis

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This paper investigates the relation between chaos and swarm intelligence. The swarm intelligent model is represented as an iterated function system (IFS). The dynamic trajectory of the particle is sensitive on the parameter values of IFS. The Lyapunov exponent and the correlation dimension were calculated and analyzed. Our preliminary research results illustrate that the performance of the swarm intelligent model depends on the sign of the maximum Lyapunov exponent. The particle swarm with a high maximum Lyapunov exponent usually achieved better performance, especially for multi-modal functions.

1 Introduction

In recent years, there has been a great interest in the relations between chaos and intelligence. Previous studies by Goldberger et al. [3], Sarbadhikari and Chakrabarty [8] illustrate that chaos has a great important influence on brain and evolutionary relationship between species etc. Recently chaotic dynamics in neural networks has also been investigated. The motivation for this research is to investigate the relation between chaos and swarm intelligence. The particle swarm provides a simple and very good case for the study. The simple swarm intelligent model helps to find optimal regions of complex search spaces through interaction of individuals in a population of particles. The model is based on a metaphor of social interaction, originally introduced as an optimization technique inspired by swarm intelligence and theory in general such as bird flocking, fish schooling and even human social behavior [5]. This paper focuses on the relationship between chaos and swarm intelligence. The particle swarm is investigated as a simple case. The swarm intelligent model is represented as an iterated function system (IFS) [9]. We simulate and analyze the dynamic trajectory of the particle based on the IFS. The Lyapunov exponent and the correlation dimension are calculated and analyzed.

2 Particle Swarm Model

A simple particle swarm model consists of a swarm of particles moving in an d-dimensional search space where the fitness f can be calculated as a certain quality measure. Each particle has a position represented by a position-vector $\mathbf{x}_i$ (i is the index of the particle), and a velocity represented by a velocity-vector $\mathbf{v}_i$. Each particle remembers its own best position so far in a vector $\mathbf{p}_i$, and its j-th dimensional value is p_{ij} . The best position-vector among the swarm so far is then stored in a vector $\mathbf{p}_g$, and its j-th dimensional value is p_{gj}. During the iteration time t, the update of the velocity from the previous velocity is determined by (1). And then the new position is determined by the sum of the previous position and the new velocity by (2).

$$v_{ij}(t+1) = wv_{ij}(t) + c_1r_1(p_{ij}(t) - x_{ij}(t)) + c_2r_2(p_{gj}(t) - x_{ij}(t)) \quad (1)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (2)$$

where r_1 and r_2 are the random numbers, uniformly distributed within the interval [0,1] for the j-th dimension of i-th particle. c_1 is a positive constant, called as coefficient of the self-recognition component, c_2 is a positive constant, called as coefficient of the social component. The variable w is called as the inertia factor, which value is typically setup to vary linearly from 1 to near 0 during the iterated processing. From (1), a particle decides where to move next, considering its own experience, which is the memory of its best past position, and the experience of its most successful particle in the swarm. In the particle swarm model, the particle searches the solutions in the problem space with a range $[-s, s]$ (If the range is not symmetrical, it can be translated to the corresponding symmetrical range.) In order to guide the particles effectively in the search space, the maximum moving distance during one iteration must be clamped in between the maximum velocity $[-x_{max}, x_{max}]$ given in (3), and similarly for its moving range given in (4):

$$v_{ij} = sign(x_{ij})min(|x_{ij}|, x_{max}) \quad (3)$$

$$v_{ij} = sign(v_{ij})min(|v_{ij}|, v_{max}) \quad (4)$$

The value of v_{max} is $\rho \times s$, with $0.1 \leq \rho \leq 1.0$ and is usually chosen to be s, i.e. $\rho = 1$. The main pseudo-code for particle-searching is listed in Algorithm 1.

3 Iterated Function System and its Sensitivity

Clerc and Kennedy have stripped the algorithm down to a most simple form [2]. If the self-recognition component c_1 and the coefficient of the social-recognition component c_2 are combined into a single term c, i.e. $c = c_1 + c_2$, the equation can be shortened by redefining $\mathbf{p}_i$ as follows:

Algorithm 1 Particle Swarm Model

Initialize the size of the particle swarm n, and other parameters.
Initialize the positions and the velocities for all the particles randomly.
While (the end criterion is not met) do
    $t = t + 1$;
    Calculate the fitness value of each particle;
    $\mathbf{p}_g(t) = argmin_{i=1}^{n}(f(\mathbf{p}_g(t-1)), f(\mathbf{x}_1(t)), f(\mathbf{x}_2(t)), \cdots, f(\mathbf{x}_i(t)), \cdots, f(\mathbf{x}_n(t)))$;
    For i= 1 to n
        $\mathbf{p}_i(t) = argmin_{i=1}^{n}(f(\mathbf{p}_i(t-1)), f(\mathbf{x}_i(t))$;
        For j = 1 to *Dimension*
            Update the j-th dimension value of $\mathbf{x}_i$ and $\mathbf{v}_i$
              according to (1), (4), (2), (3);
        Next j
    Next i
End While.

$$\mathbf{p}_i \leftarrow \frac{(c_1\mathbf{p}_i + c_2\mathbf{p}_g)}{(c_1 + c_2)} \tag{5}$$

Then the update of the particle's velocity is defined by:

$$\mathbf{v}_i(t+1) = \mathbf{v}_i(t) + c(\mathbf{p}_i - \mathbf{x}_i(t)) \tag{6}$$

The system can be simplified even further by using $\mathbf{y}_i(t)$ instead of $\mathbf{p}_i - \mathbf{x}_i(t)$. Thus we begin with single particle by considering the reduced system:

$$\begin{cases} \mathbf{v}(t+1) = \mathbf{v}(t) + c\mathbf{y}(t) \\ \mathbf{y}(t+1) = -\mathbf{v}(t) + (1-c)\mathbf{y}(t) \end{cases}$$

This recurrence relation can be written as a matrix-vector product, so that

$$\begin{bmatrix} \mathbf{v}(t+1) \\ \mathbf{y}(t+1) \end{bmatrix} = \begin{bmatrix} 1 & c \\ -1 & 1-c \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}(t) \\ \mathbf{y}(t) \end{bmatrix}$$

Let

$$\mathbf{P}_t = \begin{bmatrix} \mathbf{v}_t \\ \mathbf{y}_t \end{bmatrix}$$

and

$$A = \begin{bmatrix} 1 & c \\ -1 & 1-c \end{bmatrix}$$

we have an iterated function system for PSO:

$$\mathbf{P}_{t+1} = A \cdot \mathbf{P}_t$$

Thus the system is completely defined by A. Its norm $\|A\| > 1$ is determined by c. The varying curve of A dependent on c is illustrated in Figure 1. Considering the IFS, we discuss the particle swarm model with c in the interval [0.5,

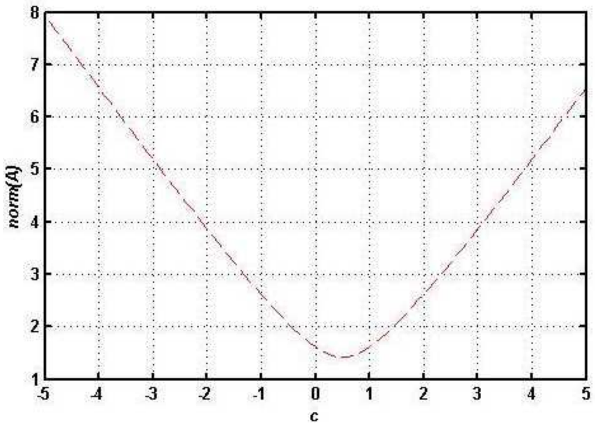

Fig. 1. Norm of A for varying values for c

4], i.e. $1.4142 < \|A\| < 5.1926$. IFS is sensitive to the values of c. We can find different trajectories of the particle for various values of c. Figure 2(a) illustrates the system for a torus when c=2.99; Figure 2(b), a hexagon with spindle sides when c=2.999; Figure 2(c), a triangle with spindle sides when c=2.9999; Figure 2(d), a simple triangle when c=2.99999. As depicted in Figure 2, the iteration time-step used is 2000 for all the cases.

4 Dynamic Chaotic Characteristics

Chaotic dynamics is defined by a deterministic system with non-regular, chaotic behavior [7]. They are both sensitive to initial conditions and computational unpredictability. The Lyapunov exponent and correlation dimension are most accessible in numerical computations based on the time-series of the dynamical system [6]. In this section, we introduce the algorithm to compute the Lyapunov exponent and correlation dimension for quantitative observation of dynamic characteristics of the particles, and then analyze the relation between chaos and the swarm intelligent model.

4.1 Lyapunov Exponent

Lyapunov exponents provide a way to identify the qualitative dynamics of a system. This is because they describe the rate at which neighboring trajectories converge or diverge (if negative or positive, respectively) from one another in orthogonal directions. If the dynamics occur in an n-dimensional system, there are n exponents. The sum of the Lyapunov exponents is the rate of system expansion. Since chaos can be defined as divergence between

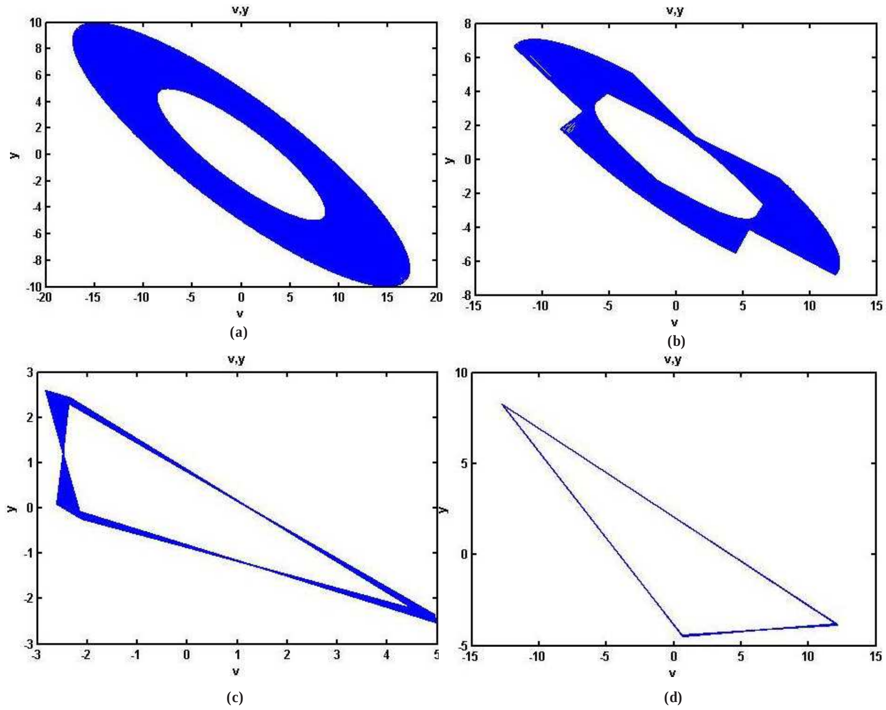

Fig. 2. Trajectory of the particle (a) $c = 2.99$, (b) $c = 2.999$, (c) $c = 2.9999$, (d) $c = 2.99999$.

neighboring trajectories, the presence of a positive exponent is the diagnostic of chaos. For an IFS, Lyapunov exponents measure the asymptotic behavior of tangent vectors under iteration. The maximum Lyapunov exponent can be found using [11]:

$$\lambda_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} log_2(\frac{d_n}{d_1}) \tag{7}$$

Where d_n is the distance between the n-th point-pair. λ_1 can be calculated using a programmable calculator to a reasonable degree of accuracy by choosing a suitably large "N". We calculated the maximum Lyapunov exponent of the IFS and is illustrated in Figure 3. The maximum Lyapunov exponent steadily increases with the value of c in the interval [0.5, 4].

4.2 Correlation Dimension

The dimension in a chaotic system is a measure of its geometric scaling property or its "complexity" and has been considered as the most basic property.

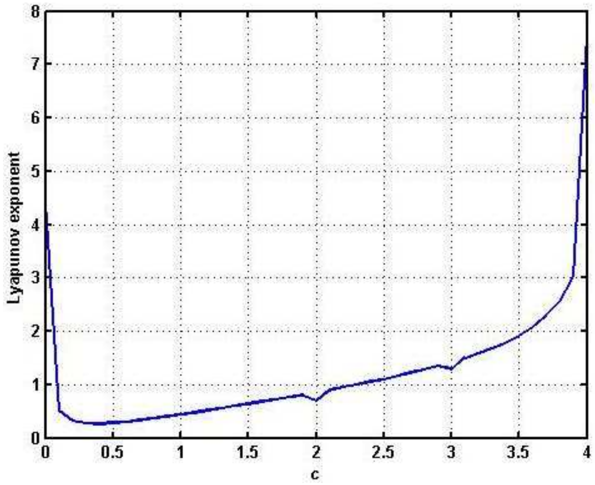

Fig. 3. Lyapunov exponent in PSO

Numerous methods have been proposed for characterizing the dimension produced by chaotic flows. The most common metrics is the correlation dimension, popularized by Grassberger and Procaccia [4]. During the past decades, several investigators have undertaken nonlinear analysis using Grassberger and Procaccia's algorithm to evaluate the correlation dimension of time-series data [10, 1].

Given by N points $\{x_1, x_2, \cdots, x_N\}$, under iteration of IFS, the correlation integral is defined by (8):

$$D = -\lim_{r \to 0} \frac{lnC(r)}{ln(r)} \tag{8}$$

In practice, $C(r)$ is calculated for several values of r and then a plot is drawn for $lnC(r)$ versus $ln(r)$ to estimate the slope, which then approximates the correlation dimension D_2. When $c = 3.9$, the slope, i.e. D_2 is illustrated in Figure 4. The correlation dimension is depicted in Figure 5. There are no obvious differences for c values increasing in the interval [0.5, 4]. D_2 is fluctuating mainly within 1 ± 0.2.

4.3 Discussions

For analyzing the relation between chaos and the swarm intelligent model, we optimized two unconstrained real-valued benchmark functions, and then investigated contrastively the performance of the model with the dynamic chaotic characteristics. One is the sphere function, given in (9). It is a continuous, unimodal function, $\mathbf{x}^* = (0, \cdots, 0)$, with $f(\mathbf{x}^*) = 0$. The other is the

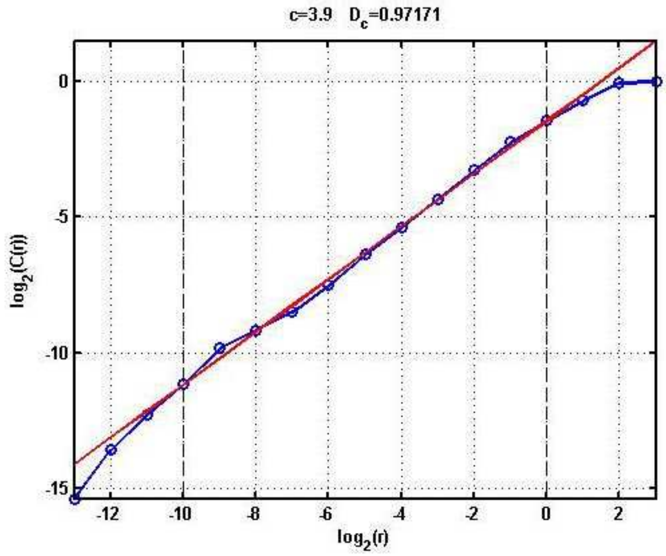

Fig. 4. $lnC(r) - ln(r)$ curve ($c = 3.9$)

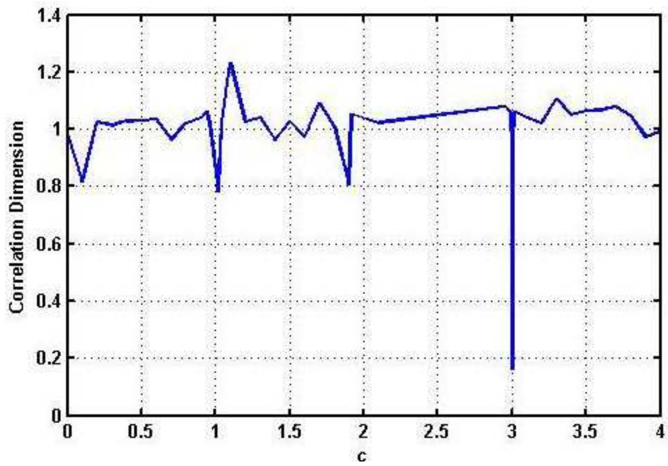

Fig. 5. Correlation dimension for varying values of c

Rastrigin's function, given by (10). It is a continuous, multimodal function with multiple local minima. And it has a "large scale" curvature which guides the search towards the global minimum, $\mathbf{x}^* = (0, \cdots, 0)$, with $f(\mathbf{x}^*) = 0$.

$$f(\mathbf{x}) = \sum_{n=1}^{n} x_i^2 \tag{9}$$

$$f(\mathbf{x}) = \sum_{n=1}^{n} [x_i^2 - 10cos(2\pi x_i) + 10] \tag{10}$$

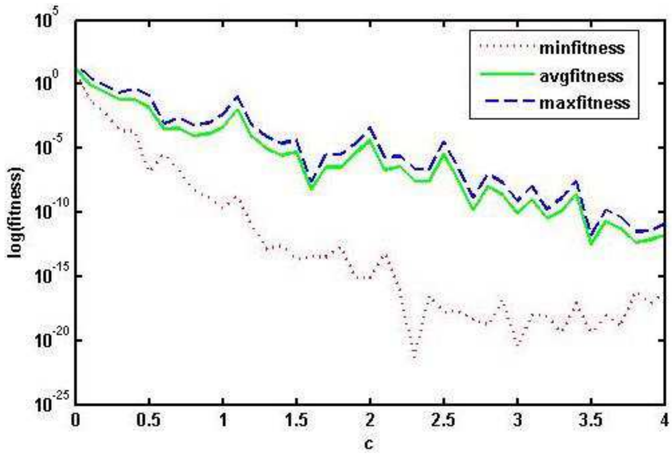

Fig. 6. The performance curve for varying values of c for 5-D Sphere function

For the two functions, the goal of particle swarm is to find the global minimum. In our experiments, V_{max} and X_{max} are set to 5.12. All experiments for both functions were run 10 times, and the maximum fitness (maxfitness), minimum fitness (minfitness) and the average fitness (avgfitness) were recorded. The swarm size is set at 10, and 200 iterations for sphere function and the result are illustrated in Figure 6. The swarm size is set at 20, and 2000 iterations for Rastrigin's function and the results are illustrated in Figure 7. Compared to the results showed in Figure 3, it is obvious that the particle swarm with a high maximum Lyapunov exponent usually achieved better performance, especially for the multi-modal functions, as showed in Figure 7. The positive Lyapunov exponent describes the rate at which neighboring trajectories diverge. A high Lyapunov exponent in the particle swarm system implies that the particles are inclined to explore different regions and find the better fitness values. Since the dimension of the particle swarm is determined by the

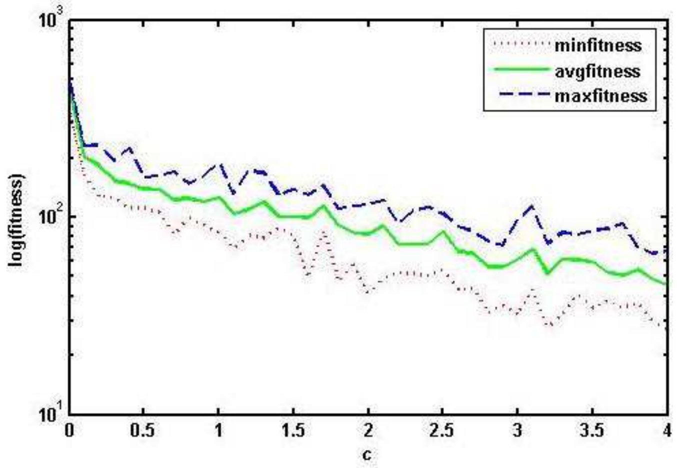

Fig. 7. The performance curve for varying values of c for 5-D Rastrigrin function

objective function, there is no significant difference in the correlation dimension. The explicit relation between correlation dimension and the performance of particle swarm can not be found in our experiments. It certainly deserves some further study.

5 Conclusions and Future Work

In this paper, we focused on the relation between chaos and swarm intelligence. The particle swarm was investigated as a simple case and the swarm model was represented by iterated function system (IFS). The dynamic trajectory of the particle was sensitive on the value of the IFS parameters. We introduced the algorithm to compute the Lyapunov exponent and correlation dimension for quantitative observation of dynamic characteristics of the particles, and then analyzed the relation between chaos and the swarm intelligent model. The results illustrated the performance of the swarm intelligent model depended on the sign of the maximum Lyapunov exponent. The particle swarm with a high maximum Lyapunov exponent usually achieved better performance, especially for the multi-modal functions.

It is noted that the real intelligent model is more complex than one which we investigated in the present paper. But it provided more aspects for further research on swarm intelligence. There are at least two works for future: 1) We could introduce chaos to overcome the problem of premature convergence in PSO, which would enjoy the ergodicity, stochastic behavior, and regularity of chaos to lead particles' exploration. Taking advantage of this characteristic feature of the chaotic system, more efficient approaches for maintaining the population diversity could be designed for some interesting problems. 2) We could design more iterated function systems to construct better models or algorithms.

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